Late-Time Evolution of Realistic Rotating Collapse and The No-Hair Theorem

Shahar Hod

The Racah Institute for Physics, The Hebrew University, Jerusalem 91904, Israel

(February 7, 2008)

Abstract

We study *analytically* the asymptotic late-time evolution of realistic *rotating* collapse. This is done by considering the asymptotic late-time solutions of Teukolsky's master equation, which governs the evolution of gravitational, electromagnetic, neutrino and scalar perturbations fields on Kerr spacetimes. In accordance with the *no-hair conjecture* for rotating black-holes we show that the asymptotic solutions develop inverse *power-law* tails at the asymptotic regions of timelike infinity, null infinity and along the black-hole outer horizon (where the power-law behaviour is multiplied by an oscillatory term caused by the dragging of reference frames). The damping exponents characterizing the asymptotic solutions at timelike infinity and along the black-hole outer horizon are independent of the spin parameter of the fields. However, the damping exponents at future null infinity are *spin dependent.* The late-time tails at all the three asymptotic regions are spatially dependent on the spin parameter of the field. The *rotational* dragging of reference frames, caused by the rotation of the black-hole (or star) leads to an active *coupling* of different multipoles.

I. INTRODUCTION

The *no-hair theorem*, introduced by Wheeler in the early 1970s [1], states that the external field of a black-hole relaxes to a Kerr-Newman field characterized solely by the black-hole's mass, charge and angular-momentum. The mechanism for this relaxation process of neutral fields was first analyzed by Price [2] for a nearly spherical collapse. The physical mechanism by which a charged black-hole, which is formed during a gravitational collapse of a *charged* matter, dynamically sheds its charged hair was first studied in [3,4]. However, these analysis were restricted to spherically symmetric backgrounds, i.e., to the Schwarschild and Reissner-Nordström black-holes. On the other hand, the physical process of stellar core collapse to form a black-hole is expected to be *non*-spherical in nature because of stellar rotation. The analogous problem of the dynamics of massless perturbations outside realistic, *rotating* black-holes is much more complicated due to the lack of spherical symmetry.

Recently, this problem was addressed numerically [5,6] and analytically for the case of scalar-perturbations [7]. However, the evolution of higher-spin fields (and in particular *gravitational* perturbations) outside realistic, rotating black-holes was not analyzed analytically so far. In addition, the works done so far were restricted to the asymptotic regions of timelike infinity [5–7] and along the black-hole outer-horizon [7]. The asymptotics at null infinity of rotating collapse was not studied so far. In this paper we give for the first time a full analytic analysis of the late-time evolution of realistic rotating collapse. Our analysis considers massless fields with arbitrary spin (and in particular *gravitational* perturbations). In addition, we consider the late-time evolution at *all* the three asymptotic regions: timelike infinity, future null infinity and along the black-hole outer horizon.

There are two different approaches to the study of perturbations of Kerr spacetimes. The first is to consider perturbations of the metric functions. However, this direct approach leads to gauge-dependent formulations. An alternative approach is to consider *curvature* perturbations (perturbations of the Weyl scalars). Based on the tetrad formalism by Newman and Penrose, Teukolsky derived a master equation governing the perturbations of Kerr

spacetimes [8,9]. The main goal of this paper is to analyze the asymptotic *late*-time evolution of a realistic *rotating* collapse. This is done by studying the asymptotic late-time solutions of the Teukolsky equation.

The plan of the paper is as follows. In Sec. II we give a short description of the physical system and formulate the evolution equation considered. In Sec. III we formulate the problem in terms of the black-hole Green's function using the technique of spectral decomposition. In Sec. IV we study the late-time evolution of perturbations of Kerr spacetimes, i.e., the asymptotic late-time solutions of the Teukolsky equation. We conclude in Sec. V with a brief summary of our results.

II. DESCRIPTION OF THE SYSTEM

We consider the evolution of gravitational, electromagnetic, neutrino and scalar (massless) perturbations fields outside a *rotating* collapsing star. The external gravitational field of a rotating object of mass M and angular-momentum per unit-mass a is given by the Kerr metric, which in Boyer-Lindquist coordinates takes the form

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 + \left(\frac{4Marsin^2\theta}{\Sigma}\right) dtd\varphi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - sin^2\theta \left(r^2 + a^2 + \frac{2Ma^2rsin^2\theta}{\Sigma}\right) d\varphi^2 \ , \tag{1}$$

where $\Sigma = r^2 + a^2cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$. Throughout this paper we use $G = c = 1$.

Using the Newman-Penrose formalism, Teukolsky [8,9] derived a master equation that governs the evolution of perturbations of the Kerr spacetime

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2sin^2\theta\right] \frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2} - \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-m)}{\Delta} + \frac{icos\theta}{sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi} - 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - iacos\theta\right]\frac{\partial\psi}{\partial t} + (s^2cot^2\theta - s)\psi = 0 \ \ , \tag{2}$$

where the parameter s is the spin-weight of the field. The Teukolsky's master equation is valid for scalar perturbations fields ($s = 0$), neutrino perturbations fields ($s = \pm 1/2$),

electromagnetic perturbations fields ($s = \pm 1$) and gravitational perturbations ($s = \pm 2$). The field quantities ψ which satisfy this equation (for the various values of the spin parameter s) are given in [9]. Resolving the field in the form

$$\psi = \Delta^{-s/2}(r^2+a^2)^{-1/2}\sum_{m=-\infty}^{\infty}\Psi^m e^{im\varphi} \quad , \tag{3}$$

one obtains a wave-equation for each value of m

$$B_1(r,\theta)\frac{\partial^2\Psi}{\partial t^2}+B_2(r,\theta)\frac{\partial\Psi}{\partial t}-\frac{\partial^2\Psi}{\partial y^2}+B_3(r,\theta)\Psi-\frac{\Delta}{(r^2+a^2)^2}\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\Psi}{\partial\theta}\right)=0 \quad , \tag{4}$$

where the tortoise radial coordinate y is defined by $dy = \frac{r^2+a^2}{\Delta}dr$. The coefficients $B_i(r,\theta)$ are given by

$$B_1(r,\theta) = 1-\frac{\Delta a^2 sin^2\theta}{(r^2+a^2)^2} \quad , \tag{5}$$

$$B_2(r,\theta) = \left\{\frac{4iMmar}{\Delta}-2s\left[\frac{M(r^2-a^2)}{\Delta}-r-iacos\theta\right]\right\}\frac{\Delta}{(r^2+a^2)^2} \quad , \tag{6}$$

and

$$B_3(r,\theta) = \Big\{2(s+1)(r-M)\left[s(r-M)\Delta^{-1}+r(r^2+a^2)^{-1}\right]-m^2\left[\frac{a^2}{\Delta}-\frac{1}{sin^2\theta}\right]- \\ 2sim\left[\frac{a(r-M)}{\Delta}+\frac{icos\theta}{sin^2\theta}\right]+(s^2cot^2\theta-s)\Big\}\frac{\Delta}{(r^2+a^2)^2} \quad . \tag{7}$$

III. FORMALISM

The time-evolution of a wave-field described by Eq. (4) is given by

$$\Psi(z,t) = 2\pi\int\int_0^\pi\Big\{B_1(z')\Big[G(z,z';t)\Psi_t(z',0)+G_t(z,z';t)\Psi(z',0)\Big]+ \\ B_2(z')G(z,z';t)\Psi(z',0)\Big\}sin\theta' d\theta' dy' \quad , \tag{8}$$

for $t>0$, where z stands for (y,θ). The (retarded) Green's function $G(z,z';t)$ is defined by

$$\left[B_1(r,\theta)\frac{\partial^2}{\partial t^2}+B_2(r,\theta)\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}+B_3(r,\theta)-\frac{\Delta}{(r^2+a^2)^2}\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right)\right]G(z,z';t) = \\ \delta(t)\delta(y-y')\frac{\delta(\theta-\theta')}{2\pi sin\theta} \quad . \tag{9}$$

The causality condition gives us the initial condition $G(z, z'; t) = 0$ for $t \leq 0$. In order to find $G(z, z'; t)$ we use the Fourier transform

$$\tilde{G}_l(y, y'; w)_s S_l^m(\theta', aw) = 2\pi \int_{0^-}^{\infty} \int_0^{\pi} G(z, z'; t)_s S_l^m(\theta, aw) sin\theta e^{iwt} d\theta dt \quad , \tag{10}$$

where ${}_sS_l^m(\theta, aw)$ are the spin-weighted spheroidal harmonics which are solutions to the angular-equation [9]

$$\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \left(a^2w^2cos^2\theta - \frac{m^2}{sin^2\theta} - 2awscos\theta - \frac{2mscos\theta}{sin^2\theta} - s^2cot^2\theta + s + {}_sA_l^m\right) {}_sS_l^m = 0 \ . \tag{11}$$

For the $aw = 0$ case, the eigenfunctions ${}_sS_l^m(\theta, aw)$ reduce to the spin-weighted spherical harmonics ${}_sY_l^m(\theta, \phi) = {}_sS_l^m(\theta)e^{im\varphi}$, and the separation constants ${}_sA_l^m(aw)$ are simply ${}_sA_l^m = (l-s)(l+s+1)$ [10].

The Fourier transform is analytic in the upper half w-plane and it satisfies the equation [9]

$$\left\{\frac{d^2}{dy^2} + \left[\frac{K^2 - 2is(r-M)K + \Delta(4irws - \lambda)}{(r^2+a^2)^2} - H^2 - \frac{dH}{dy}\right]\right\}\tilde{G}_l(y, y'; w) = \delta(y - y') \ , \tag{12}$$

where $K = (r^2+a^2)w - am$, $\lambda = A + a^2w^2 - 2amw$ and $H = s(r-M)/(r^2+a^2) + r\Delta/(r^2+a^2)^2$. $G(z, z'; t)$ itself is given by the inversion formula

$$G(z, z'; t) = \frac{1}{(2\pi)^2}\sum_{l=l_0}^{\infty}\int_{-\infty+ic}^{\infty+ic}\tilde{G}_l(y, y'; w)_s S_l^m(\theta, aw)_s S_l^m(\theta', aw)e^{-iwt}dw \ , \tag{13}$$

where c is some positive constant and $l_0 = max(|m|, |s|)$.

Next, we define two auxiliary functions $\tilde{\Psi}_1(z, w)$ and $\tilde{\Psi}_2(z, w)$ which are (linearly independent) solutions to the homogeneous equation

$$\left[\frac{d^2}{dy^2} + \frac{K^2 - 2is(r-M)K + \Delta(4irws - \lambda)}{(r^2+a^2)^2} - H^2 - \frac{dH}{dy}\right]\tilde{\Psi}_i(y, w) = 0 \ , \quad i = 1, 2 \ . \tag{14}$$

The two basic solutions that are required in order to build the black-hole Green's function are defined by their asymptotic behaviour:

$$\tilde{\Psi}_1(y,w) \sim \begin{cases} \Delta^{-s/2} e^{-iky} & , \quad y \to -\infty \ , \\ A_{out}(w) y^{i2wM-s} e^{iwy} + A_{in}(w) y^{-i2wM+s} e^{-iwy} & , \quad y \to \ \infty \ , \end{cases} \tag{15}$$

and

$$\tilde{\Psi}_2(y,w) \sim \begin{cases} B_{out}(w) \Delta^{s/2} e^{iky} + B_{in}(w) \Delta^{-s/2} e^{-iky} & , \quad y \to -\infty \ , \\ y^{i2wM-s} e^{iwy} & , \quad y \to \infty \ , \end{cases} \tag{16}$$

where $k = w - mw_+$, $w_+ = a/(2Mr_+)$. Let the Wronskian be

$$W(w) = W(\tilde{\Psi}_1, \tilde{\Psi}_2) = \tilde{\Psi}_1 \tilde{\Psi}_{2,y} - \tilde{\Psi}_2 \tilde{\Psi}_{1,y} \ , \tag{17}$$

where $W(w)$ is y-independent. Thus, using the two solutions $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$, the black-hole Green's function can be expressed as

$$\tilde{G}_l(y,y';w) = -\frac{1}{W(w)} \begin{cases} \tilde{\Psi}_1(y,w)\tilde{\Psi}_2(y',w) & , \quad y < y' \ , \\ \tilde{\Psi}_1(y',w)\tilde{\Psi}_2(y,w) & , \quad y > y' \ . \end{cases} \tag{18}$$

In order to calculate $G(z,z';t)$ using Eq. (13), one may close the contour of integration into the lower half of the complex frequency plane. Then, one finds three distinct contributions to $G(z,z';t)$ [11] :

1. *Prompt contribution.* This comes from the integral along the large semi-circle. This term contributes to the *short*-time response.

2. *Quasinormal modes.* This comes from the distinct singularities of $\tilde{G}(y,y';w)$ in the lower half of the complex w-plane, and it is just the sum of the residues at the poles of $\tilde{G}(y,y';w)$. Since each mode has Im$w < 0$ this term decays *exponentially* with time.

3. *Tail contribution.* The late-time tail is associated with the existence of a branch cut (in $\tilde{\Psi}_2$) [11] , usually placed along the negative imaginary w-axis. This tail arises from the integral of $\tilde{G}(y,y';w)$ around the branch cut and is denoted by G^C. As will be shown, the contribution G^C leads to an inverse *power-law* behaviour (multiplied by a periodic term along the black-hole outer horizon) of the field. Thus, G^C dominates the late-time behaviour of the field.

The present paper focuses on the late-time asymptotic solutions of the Teukolsky equation. Thus, the purpose of this paper is to evaluate $G^C(z,z';t)$.

IV. LATE-TIME BEHAVIOUR OF REALISTIC ROTATING COLLAPSE

A. The large-r (low-w) approximation

It is well known that the late-time behaviour of massless perturbations fields is determined by the backscattering from asymptotically *far* regions [12,2]. Thus, the late-time behaviour is dominated by the *low*-frequencies contribution to the Green's function, for only low frequencies will be backscattered by the small effective potential (for $r \gg M$) in Eq. (14). Thus, as long as the observer is situated far away from the black-hole and the initial data has a considerable support only far away from the black-hole, a *large*-r (or equivalently, a *low*-w) approximation is sufficient in order to study the asymptotic *late-time* behaviour of the field [13]. Expanding Eq. (14) for large r one obtains [using $\lambda=(l-s)(l+s+1)+O(aw)$ and neglecting terms of order $O(\frac{w}{r^2})$ and higher]

$$\left[\frac{d^2}{dr^2}+w^2+\frac{4Mw^2+2isw}{r}-\frac{l(l+1)}{r^2}\right]\tilde{\Psi}=0\ . \tag{19}$$

It should be noted that this equation is the *correct* form of equation (66) of Ref. [13] (the term s^2/r^2 should *not* appear in that equation).

We introduce a second auxiliary field $\tilde{\phi}$ defined by

$$\tilde{\Psi}=r^{l+1}e^{iwr}\tilde{\phi}(x)\ , \tag{20}$$

where $x=-2iwr$. $\tilde{\phi}(x)$ satisfies the confluent hypergeometric equation

$$\left[x\frac{d^2}{dx^2}+(2l+2-x)\frac{d}{dx}-(l+1-2iw\alpha)\right]\tilde{\phi}(x)=0\ , \tag{21}$$

where

$$\alpha=M+\frac{is}{2w}\ . \tag{22}$$

Thus, the two basic solutions required in order to build the Green's function are (for $r \gg M, |a|$)

$$\tilde{\Psi}_1 = Ar^{l+1} e^{iwr} M(l+s+1-2iwM, 2l+2, -2iwr) \ , \tag{23}$$

and

$$\tilde{\Psi}_2 = Br^{l+1} e^{iwr} U(l+s+1-2iwM, 2l+2, -2iwr) \ , \tag{24}$$

where A and B are normalization constants. $M(a,b,z)$ and $U(a,b,z)$ are the two standard solutions to the confluent hypergeometric equation [14]. From these solutions it is clear that the black-hole's rotation parameter a is irrelevant in the context of the *asymptotic* form of the Green's function (to leading order in the inverse time). However, as will be shown below, the *rotational* dragging of reference frames, caused by the rotation of the black-hole (or star), has (two) important effects (which are absent in the non-rotating case) on the asymptotic late-time evolution of *rotating* collapse. Hence, in order to find the asymptotic form of the Green's function it is sufficient to analyze the late-time solutions in the *spherically* symmetric limit (i.e, on the Schwarchild background).

A note is needed here on the relations between the various equations governing the perturbations of the Schwarchild spacetime. There are two different approaches to the study of linearized gravitational perturbations: by considering metric perturbations, or, alternatively, by considering perturbations of the Weyl scalars. Equations governing metric perturbations of the Schwarchild spacetime were derived by Regge and Wheeler [15] (for odd-parity perturbations) and by Zerilli [16] (for polar perturbations). An alternative approach, based on the tetrad formalism by Newman and Penrose was used by Teukolsky [7,8] to derive a separable wave equation for perturbations of the Weyl Scalars. These scalars, constructed from the Weyl tensor in a given tetrad basis, characterize the gravitational field in vacuum. They allow a more convenient approach to the study of gravitational perturbations, due to their scalar nature. As was first shown by Chandrasekhar [17], the $a \to 0$ limit of the Teukolsky equation, the Bardeen-Press equation [18], can easily be transformed into the

Regge-Wheeler equation. However, this equivalence is restricted to the *frequency* domain. In other wards, any solution (in the frequency domain) of the Bardeen-Press equation can be transformed in a trivial manner into a solution of the Regge-Wheeler equation, and vise versa. However, these solutions have, of coarse, a *different* w-dependence (as can be verified for example from their asymptotic forms Eqs. (23) and (24)). Hence, this difference in the w-dependence of the solutions, when integrated in the complex frequency plane (in order to obtain the *temporal* dependence of the solutions) may lead to a non-trivial difference in the time dependence of the solutions. Thus, it is not apriori guaranteed that the asymptotic late-time solutions of Teukolsky's equation and the Regge-Wheeler equation (studied by price [2] for the spherically symmetric case) will have the same temporal behaviour (Indeed, it is found that the damping exponents, describing the fall-off of the asymptotic solutions at future null infinity are spin-dependent, a phenomena not found for the Regge-Wheeler asymptotic solutions, see Sec. IV C below). Furthermore, one can easily verify that even the $a \to 0$ limit of the Teukolsky equation depends on the spin parameter s [see Eq. (2) and its asymptotic form Eq. (19)]. Hence, it is not surprising that the asymptotic late-time solutions of the Teukolsky equation will turn out to be spin-dependent.

The function $U(a,b,z)$ is a many-valued function, i.e., there will be a cut in $\tilde{\Psi}_2$. Using Eq. (13), one finds that the branch cut contribution to the Green's function is given by

$$G^C(z,z';t) = \frac{1}{(2\pi)^2} \sum_{l=l_0}^{\infty} \int_0^{-i\infty} \tilde{\Psi}_1(y',w) \left[\frac{\tilde{\Psi}_2(y,we^{2\pi i})}{W(we^{2\pi i})} - \frac{\tilde{\Psi}_2(y,w)}{W(w)} \right] {}_sS_l(\theta,aw)\,{}_sS_l(\theta',aw)e^{-iwt}dw \ . \tag{25}$$

(For simplicity we assume that the initial data has a considerable support only for r-values which are smaller than the observer's location. This, of course, does not change the asymptotic *late*-time behaviour).

Using the fact that $M(a,b,z)$ is a single-valued function and Eq. 13.1.10 of [14] (taking the $b \to k$ limit, where k is an integer), one finds

$$\tilde{\Psi}_1(r,we^{2\pi i}) = \tilde{\Psi}_1(r,w) \ , \tag{26}$$

and

$$\tilde{\Psi}_2(r,we^{2\pi i})=\tilde{\Psi}_2(r,w)+\frac{B}{A}\frac{(-1)^{2l}2\pi i}{(2l+1)!\Gamma(-l+s-2iwM)}\tilde{\Psi}_1(r,w)\ . \tag{27}$$

Using Eqs. (26) and (27) it is easy to see that

$$W(we^{2\pi i})=W(w)\ . \tag{28}$$

Thus, using Eqs. (26), (27) and (28), we obtain the relation

$$\frac{\tilde{\Psi}_2(r,we^{2\pi i})}{W(we^{2\pi i})}-\frac{\tilde{\Psi}_2(r,w)}{W(w)}=\frac{B}{A}\frac{(-1)^{2l}2\pi i}{(2l+1)!\Gamma(-l+s-2iwM)}\frac{\tilde{\Psi}_1(r,w)}{W(w)}\ . \tag{29}$$

Since $W(w)$ is r-independent, we may use the large-r asymptotic expansions of the confluent hypergeometric functions (given by Eqs. 13.5.1 and 13.5.2 in [14]) in order to evaluate it. One finds

$$W(w)=i\frac{AB(-1)^{l+1}(2l+1)!w^{-2l-1}}{\Gamma(l+s+1-2iwM)2^{2l+1}}\ . \tag{30}$$

(Of coarse, using the $|z|\to 0$ limit of the confluent hypergeometric functions, we obtain the same result). Thus, substituting Eqs. (29) and (30) in Eq. (25) one finds

$$G^C(z,z';t)=\sum_{l=l_0}^{\infty}\frac{(-1)^{l+1}2^{2l}}{\pi A^2[(2l+1)!]^2}\int_0^{-i\infty}\frac{\Gamma(l+s+1-2iwM)}{\Gamma(-l+s-2iwM)}$$
$$\tilde{\Psi}_1(y,w)\tilde{\Psi}_1(y',w)_sS_l(\theta,aw)_sS_l(\theta',aw)w^{2l+1}e^{-iwt}dw\ . \tag{31}$$

As was explained, the late-time behaviour of the field should follow from the *low*-frequency contribution to the Green's function. It is clear that all the rotation-dependent terms of Eq. (11) can be neglected in the $aw\to 0$ limit. Hence, one may replace the functions ${}_sS_l^m(\theta,aw)$ by the spin-weighted spherical harmonics ${}_sY_l^m(\theta,\phi)$. In addition, we use the approximation

$$\frac{\Gamma(l+s+1-2iwM)}{\Gamma(-l+s-2iwM)}\simeq 2iwM(-1)^{l-s+1}(l+s)!(l-s)!\ , \tag{32}$$

which is valid for $w\to 0$. With these substitutions Eq. (31) becomes

$$G^C(z,z';t)=\sum_{l=l_0}^{\infty}\frac{iM(-1)^{2l-s}2^{2l+1}(l+s)!(l-s)!}{\pi A^2[(2l+1)!]^2}{}_sY_l(\theta)_sY_l^*(\theta')$$
$$\int_0^{-i\infty}\tilde{\Psi}_1(y,w)\tilde{\Psi}_1(y',w)w^{2l+2}e^{-iwt}dw\ . \tag{33}$$

B. Asymptotic behaviour at timelike infinity

First, we consider the asymptotic behaviour of the fields at *timelike infinity* i_+. As was explained, the late-time behaviour of the field should follow from the *low*-frequency contribution to the Green's function. Actually, it is easy to verify that the effective contribution to the integral in Eq. (33) should come from $|w|=O(\frac{1}{t})$. Thus, in order to obtain the asymptotic behaviour of the field at *timelike infinity* (where $y, y' \ll t$), we may use the $|w|r \ll 1$ limit of $\tilde{\Psi}_1(r,w)$. Using Eq. 13.5.5 from [14] one finds

$$\tilde{\Psi}_1(r,w) \simeq Ar^{l+1} \ . \tag{34}$$

Substituting this in Eq. (33) we obtain

$$\begin{aligned} G^C(z,z';t) = \sum_{l=l_0}^{\infty} \frac{iM(-1)^{2l-s}2^{2l+1}(l+s)!(l-s)!}{\pi[(2l+1)!]^2} {}_sY_l(\theta){}_sY_l^*(\theta')(yy')^{l+1} \\ \int_0^{-i\infty} w^{2l+2}e^{-iwt}dw \ , \end{aligned} \tag{35}$$

Performing the integration in Eq. (35), one finds

$$\begin{aligned} G^C(z,z';t) = \sum_{l=l_0}^{\infty} \frac{M(-1)^{l-s+1}2^{2l+1}(l+s)!(l-s)!(2l+2)!}{\pi[(2l+1)!]^2} \\ {}_sY_l(\theta){}_sY_l^*(\theta')y'^{l+1}y^{l+1}t^{-(2l+3)} \ . \end{aligned} \tag{36}$$

C. Asymptotic behaviour at future null infinity

Next, we go on to consider the behaviour of the fields at future null infinity $scri_+$. It is easy to verify that for this case the effective frequencies contributing to the integral in Eq. (33) are of order $O(\frac{1}{u})$. Thus, for $y-y' \ll t \ll 2y-y'$ one may use the $|w|y' \ll 1$ asymptotic limit for $\tilde{\Psi}_1(y',w)$ and the $|w|y \gg 1$ ($Imw < 0$) asymptotic limit of $\tilde{\Psi}_1(y,w)$. Thus,

$$\tilde{\Psi}_1(y',w) \simeq Ay'^{l+1} \ , \tag{37}$$

and

$$\tilde{\Psi}_1(y,w) \simeq Ae^{iwy}(2l+1)! \frac{e^{-i\frac{\pi}{2}(l+s+1-2iwM)}(2w)^{-l-s-1+2iwM}y^{-s+2iwM}}{\Gamma(l-s+1+2iwM)} \ , \tag{38}$$

where we have used Eqs. 13.5.5 and 13.5.1 of [14], respectively. Integrating Eq. (33) with the aid of Eqs. (37) and (38) one finds (for $v \gg u$)

$$G^C(z,z';t) = \sum_{l=l_0}^{\infty} \frac{M(-1)^{l-s+1}2^l(l+s)!(l-s+2)!(2l+2)}{\pi(l+2)!} \\ {}_sY_l(\theta) {}_sY_l^*(\theta') y'^{l+1} v^{-s} u^{-(l-s+2)} \ . \tag{39}$$

D. Asymptotic behaviour along the black-hole outer horizon

Finally, we consider the behaviour of the fields at the black-hole outer-horizon r_+. While Eqs. (23) and (24) are (approximated) solutions to the wave-equation (14) in the $r \gg M, |a|$ case, they do not represent the solution near the horizon. As $y \to -\infty$ the wave-equation (14) can be approximated by the equation [9]

$$\tilde{\Psi}_{,yy} + \left[k^2 - \frac{2is(r_+ - M)k}{2Mr_+} - \frac{s^2(r_+ - M)^2}{(2Mr_+)^2}\right]\tilde{\Psi} = 0 \ . \tag{40}$$

Thus, we take

$$\tilde{\Psi}_1(y,w) = C(w)\Delta^{-s/2}e^{-iky} \ , \tag{41}$$

and we use Eq. (37) for $\tilde{\Psi}_1(y',w)$. In order to match the $y \ll -M$ solution with the $y \gg M$ solution we assume that the two solutions have the same temporal dependence (this assumption has been proven to be very successful for neutral [19] and charged [3] perturbations on a spherically symmetric backgrounds). In other words we take $C(w)$ to be w-independent. In this case one should replace the roles of y' and y in Eq. (33). Using Eq. (33), we obtain

$$G^C(z,z';t) = \sum_{l=l_0}^{\infty} \Gamma_0 \frac{M(-1)^{l-s+1}2^{2l+1}(l+s)!(l-s)!(2l+2)!}{\pi[(2l+1)!]^2} \\ {}_sY_l(\theta) {}_sY_l^*(\theta') \Delta^{-s/2} y'^{l+1} e^{imw_+ y} v^{-(2l+3)} \ , \tag{42}$$

where Γ_0 is a constant.

E. Coupling of different multipoles

The time-evolution of a wave-field described by Eq. (4) is given by Eq. (8). The coefficients $B_1(r,\theta)$ and $B_2(r,\theta)$ appearing in Eq. (4) depend explicitly on the angular variable θ through the *rotation* of the black-hole (no such dependence exist in the $a=0$ case). This angular dependence of the coefficients leads to an active interaction between different multipoles (characterized by different values of l). This coupling between different multipoles is physically caused by the *rotational* dragging of reference frames (due to the rotation of the black-hole). Thus, even if the initial data is characterized by a pure multipole l (i.e., by a certain spin-weighted spherical harmonic ${}_sY_l$) other multipoles would be generated dynamically during the evolution. In other words, the late-time evolution of a spin-s field is dominated by the lowest allowed multipole, i.e., by the $l=|s|$ multipole, regardless of the angular dependence of the initial-data (provided that the initial-data contains modes with $|m| \leq |s|$). This phenomena of coupling between different multipoles has been observed in numerical solutions of the Teukolsky equation [5,6] (a similar phenomena is known in the case of rotating stars [20]).

V. SUMMARY

We have studied the asymptotic late-time evolution of realistic *rotating* collapse. This was done by considering the asymptotic late-time solutions of Teukolsky's master equation which governs the evolution of gravitational, electromagnetic, neutrino and scalar (massless) perturbations fields on Kerr spacetimes.

Following the *no-hair conjecture* for rotating black-holes we have shown that the asymptotic solutions develop inverse *power-law* tails at timelike infinity, at null infinity and along the black-hole outer horizon (where the power-law behaviour is multiplied by an oscillatory term, caused by the dragging of reference frames at the event horizon).

The damping exponents, describing the fall-off of the asymptotic solutions of Teukolsky's

equation at timelike infinity and along the black-hole outer horizon are independent of the spin parameter of the field. However, we have shown that the damping exponents at future null infinity are *spin-dependent*. Moreover, the asymptotic late-time solutions of Teukolsky's equation at all the three asymptotic regions have a spatial dependence on the spin parameter of the field.

The damping exponents are *independent* of the rotation parameter a of the black-hole (or star). However, the *rotational* dragging of reference frames, caused by the rotation of the black-hole has two important effects (which are not found in the non rotating case) on the asymptotic late-time evolution of *rotating* collapse: The power-law tail along the black-hole outer horizon is multiplied by an *oscillatory* term. In addition, the rotation leads to an active *coupling* of different multipoles. Hence, the late-time evolution of a spin-s field is dominated by the lowest allowed multipole, i.e., by the $l = |s|$ multipole.

ACKNOWLEDGMENTS

This research was supported by a grant from the Israel Science Foundation.

REFERENCES

[1] R. Ruffini and J. A. Wheeler, Physics Today **24**, 30 (1971); C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation (Freeman, San Francisco 1973).

[2] R.H. Price, Phys. Rev. **D 5**, 2419 (1972).

[3] S. Hod and T. Piran, Phys. Rev. D **58**, 024017 (1998).

[4] S. Hod and T. Piran, Phys. Rev. D **58**, 024018 (1998).

[5] W. Krivan, P. Laguna and P. Papadopoulos, Phys. Rev. **D 54**, 4728 (1996).

[6] W. Krivan, P. Laguna and P. Papadopoulos and N. Andersson, Phys. Rev. **D 56**, 3395 (1997).

[7] A. Ori, Gen. Rel. Grav. **29**, Number 7, 881 (1997).

[8] S. A. Teukolsky, Phys. Rev. Lett. **29**, 1114 (1972).

[9] S. A. Teukolsky, Astrophys. J. **185**, 635 (1973).

[10] J. N. Goldberg, A. J. Macfarlane, E. T. Newman, F. Rohrlich and E. C. G. Sudarshan, J. Math. Phys. **8**, 2155 (1967).

[11] E. W. Leaver, Phys. Rev. **D 34**, 384 (1986).

[12] K. S. Thorne, p. 231 in Magic without magic: John Archibald Wheeler Ed: J.Klauder (W.H. Freeman, San Francisco 1972).

[13] N. Andersson, Phys. Rev. **D 55**, 468 (1997).

[14] M. Abramowitz and I.A. Stegun, Handbook of mathematical functions (Dover Publications, New York 1970).

[15] T. Regge and J. A. Wheeler, Phys. Rev. **108**, 1063 (1957).

[16] F. J. Zerilli, Phys. Rev. Lett. **24**, 737 (1970).

[17] S. Chandrasekhar, Proc. R. Soc. London **A 343**, 289 (1975).

[18] J. M. Bardeen and W. H. Press, J. Math. Phys. **14**, 7 (1973).

[19] C. Gundlach, R.H. Price, and J. Pullin, Phys. Rev. **D 49**, 883 (1994).

[20] Y. Kojima, Phys. Rev. **D 46**, 4289 (1992).